AP Calculus BC — Derivatives Review

1) $y = \ln(5x^4)$

$y' = \frac{20x^3}{5x^4} = \frac{4}{x}$

2) $y = e^{-4x}$

$y' = -4e^{-4x}$

3) $y = 7^{3x^2+4x}$

$y' = 7^{3x^2+4x}(6x+4)\cdot \ln 7$

4) $f(x) = \sin x \cos x$

$f'(x) = -\sin^2 x + \cos^2 x$

5) $y = x^{5x}$

$\ln y = 5x \cdot \ln x$

$\frac{1}{y}\cdot y' = 5x\left(\frac{1}{x}\right) + 5\ln x$

$y' = (5 + 5\ln x)\cdot x^{5x}$

6) $y = (x^3+2)^4(\cot x - 2x)^5$

$y' = (x^3+2)^4 \cdot 5(\cot x - 2x)^4(-\csc^2 x - 2) +$

$(\cot x - 2)^5 \cdot 4(x^3+2)^3 \cdot 3x^2$

7) $y = \sin(3x-4)$

$y' = 3\cos(3x-4)$

8) $w(x) = \tan^2(\ln(1+x))$

$w'(x) = 2\tan(\ln(1+x))\cdot \sec^2(\ln(1+x))\cdot \frac{1}{1+x}$

9) $h(x) = \ln(\sec\sqrt{x})$

$h'(x) = \dfrac{\sec\sqrt{x}\tan\sqrt{x}\cdot\frac{1}{2\sqrt{x}}}{\sec\sqrt{x}}$

10) $y = \arccos(5x^3+4x)$

$y' = \dfrac{-(15x^2+4)}{\sqrt{1-(5x^3+4x)^2}}$

11) $y = \cos(\ln 3x)$

$y' = -\sin(\ln 3x)\cdot\frac{3}{3x}$

12) $f(x) = \tan^3(4x^6-2x)$

$f'(x) = 3\tan^2(4x^6-2x)\cdot\sec^2(4x^6-2x)(24x-2)$

13) $f(x) = \csc(x^6)e^{-5x}$

$f'(x) = \csc(x^6)\cdot(-5e^{-5x}) + e^{-5x}(-\csc(x^6)\cot(x^6)\cdot 6x^5)$

14) $y = \ln(x^2+5x)$

$y' = \dfrac{2x+5}{x^2+5x}$

15) $y = \sin^2 x \qquad y' = 2\sin x\cos x$

$\left(\frac{\pi}{6}, \frac{1}{4}\right) \qquad y'\Big|_{\frac{\pi}{6}} = 2\left(\frac{1}{2}\right)\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

$T: \quad y - \frac{1}{4} = \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right)$

$N = \quad y - \frac{1}{4} = -\frac{2}{\sqrt{3}}\left(x - \frac{\pi}{6}\right)$

16) $K(x) = g(f(x)) = (\sin^2 x)^2 - 5 = \sin^4 x - 5$

$K'(x) = 4\sin^3 x\cos x$

$K'\left(\frac{\pi}{4}\right) = 4\left(\frac{\sqrt{2}}{2}\right)^3\cdot\frac{\sqrt{2}}{2} = 4\left(\frac{\sqrt{2}}{2}\right)^4$

$= 1$

17) $x^2 + y^3 = 1$

$2x + 3y^2\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{2x}{3y^2}$

$\left.\frac{dy}{dx}\right|_{(3,-2)} = -\frac{6}{12}$

$= -\frac{1}{2}$

$\frac{d^2y}{dx^2} = -\frac{(3y^2)(2) - (2x)\left(6y\frac{dy}{dx}\right)}{(3y^2)^2}$

$\frac{d^2y}{dx^2} = -\frac{(12)(2) - (6)\left(-12\left(-\frac{1}{2}\right)\right)}{(12)^2}$